Eur. Phys. J. B **47**, 541–547 (2005)
DOI: 10.1140/epjb/e2005-00362-5

THE EUROPEAN PHYSICAL JOURNAL B

Universal estimation of X^- trion binding energy in semiconductor quantum wells

R.A. Sergeev[1,a], R.A. Suris[1], G.V. Astakhov[1,2], W. Ossau[2], and D.R. Yakovlev[1,3]

[1] A.F. Ioffe Physico-Technical Institute, Russian Academy of Sciences, 194021, St. Petersburg, Russia
[2] Physikalisches Institut der Universität Würzburg, 97074 Würzburg, Germany
[3] Experimentelle Physik 2, Universität Dortmund, 44221 Dortmund, Germany

Received 26 May 2005
Published online 17 November 2005 –

Abstract. We have analyzed the binding energy (E_B^T) of negatively charged excitons (X^-) in GaAs, CdTe and ZnSe quantum wells, which differ considerably in exciton and trion binding energy. Surprisingly, the E_B^T in these materials plotted against quantum well width in Bohr units is found to group around one universal curve described by a simple phenomenological equation. An illustrative model is suggested to calculate the binding energy E_B^T in a general case, and the results of calculations are in agreement with experimental data. The E_B^T dependencies on the mass ratio and the barriers height are also obtained from the general model and compared with other calculations available.

PACS. 71.35.Pq Charged excitons (trions) – 71.35.-y Excitons and related phenomena – 71.10.Ca Electron gas, Fermi gas

1 Introduction

The first consideration of an atomic-like three-body system is regarded to Bethe, by whom the attention to the hydrogen ion H^- has been attracted as early as 1929 [1]. The existence of negatively (*eeh*) and positively (*ehh*) charged excitons (trions) in semiconductors, being analog to hydrogen ions, was predicted by Lampert in 1958 [2]. The investigation of three-body complexes has a fundamental importance, particularly in semiconductors, where there is a possibility to vary parameters in a wide spectrum. However, the experimental observation of trions in bulk semiconductors is rather difficult due to their small binding energies.

The interest in experiment and theory of trions has grown due to the progress in the semiconductor heterostructure fabrication. Theoretical calculations performed at the end of the 1980s [3] predicted a considerable (up to tenfold) increase of the trion binding energy in quantum well heterostructures compared with bulk semiconductors. The first experimental observation of negatively charged excitons (X^-) has been reported for CdTe-based quantum wells (QWs) by K. Kheng et al. in 1993 [4]. The trions have also been observed in QWs based on GaAs and ZnSe semiconductors [5–7]. Nowadays, a large set of experimental data on X^- trion are available for various types of heterostructures with different parameters.

The main characteristics of the negatively (or positively) charged excitons are their binding energies, i.e. the energy required to separate the trion in a neutral exciton and an unbound electron (hole). The variation of the binding energy of X^- trion [8–11] and, the similar system, D^- center [12–14] with the QW width have been extensively studied theoretically. But, most of these calculations are limited to specific material systems. In order to achieve a better agreement with experimental data the problem is treated with a considerable number of fitting parameters.

Our aim is to compare the experimental values of the trion binding energy (E_B^T) in heterostructures based on different semiconductors with the results of a simple but universal theoretical model. Such general approach allows to render the sensitivity of the trion parameters to the specifics of real heterostructure. In the paper we concentrate on the negatively charged exciton, because of the reliable set of experimental data available. It is important to note, that, commonly, the effective mass of a hole is larger than that of an electron. So the X^- is constructed of one heavy particle only, which simplifies the theoretical consideration.

In Section 2, the experimental data for the trion binding energy in heterostructures of different material systems are summarized and discussed. In Section 3, a simple common model of the trion with a heavy hole in an ideal QW is proposed and the binding energy dependence versus the effective well width is variationally calculated. In Section 4 the deviations between the common model

[a] e-mail: rinat.theory@mail.ioffe.ru

Table 1. The original experimental data collected for various semiconductor materials. Note, that for a correct comparison the binding energy of an "isolated" trion, which is unperturbed by interaction with two-dimensional electron gas, must be taken into account [15,16]. Therefore, we either select the data for undoped QWs or extrapolate the binding energies in doped structures to the low-concentration limit. In the latter case the initial values are given in brackets. The data are taken from the following publications: [a]reference [16], [b]reference [17], [c]reference [18], [d]reference [15], [e]reference [19], [f]reference [20], [g]reference [21], [h]reference [22], [i]reference [23], [j]reference [24], [k]reference [25], [l]unpublished.

ZnSe	L_z, Å	29	48	50	64	67	80	95	190	200		
	E_B^T, meV	8.9[a]	6.6[a]	5.8[a]	5.2[a]	5.3[a]	4.4[a]	4.0[a]	1.4[a]	2.5[b]		
CdTe	L_z, Å	38	50	55	80	100	120	150	260	400	500	600
	E_B^T, meV	4.4[c]	3.5[c]	3.4[c]	2.9[c]	2.1[d]	2.5[e]	2.2[c]	1.8[c]	1.3[f]	1.2[l]	1.1[f]
						(2.6)				(1.8)		(1.5)
GaAs	L_z, Å	80	100	200	220	250	300					
	E_B^T, meV	2.1[g]	2.1[h]	1.15[i]	1.1[j]	0.8[k]	0.9[j]					

and the experimental data are discussed. The last section contains the conclusions.

2 Experimental results

Charged excitons in various heterostructures have been extensively studied during the last decade. Experimental data of the X^- trion binding energy (E_B^T) for ZnSe, CdTe and GaAs quantum wells of various widths (L_z) are collected in Table 1 and illustrated in Figure 1a. One can clearly follow the increase of the trion binding energy by decreasing well width for all types of heterostructure material.

In order to compare experimental data for different materials, we replotted them in 3D Bohr units, E_B^T/Ry against L_z/a_B, as shown in Figure 1b. In the published literature there is some dispersion (up to 15%) in the values of bulk exciton Rydberg and Bohr radius. We took the averaged ones: 3D exciton Rydberg $Ry = 4.2$, 10, 20 meV and 3D exciton Bohr radius $a_B = 140, 67, 40$ Å for GaAs, CdTe and ZnSe respectively. It can be seen that, in these units, the experimental points group around a common curve. The relative discrepancy between the results from different semiconductors does not exceed 20% for the majority of the data, whereas the binding energy itself changes by about one order of magnitude. It means, that the plausible estimation of the trion binding energy can be obtained with some universal formula. For example, the simplest fitting equation found for the quantum wells of a thicknesses between one a_B and $10a_B$ is (shown in Fig. 1b by a solid line):

$$\frac{E_B^T}{Ry} \approx \frac{1}{3\sqrt{\frac{L_z}{a_B}}}. \qquad (1)$$

Of course, this equation cannot be used for the limiting cases $L_z \to 0$ and $L_z \to \infty$. Nevertheless, due to its simplicity, equation (1) can be very useful to predict the trion binding energies in any semiconductor QW for the wide range of L_z.

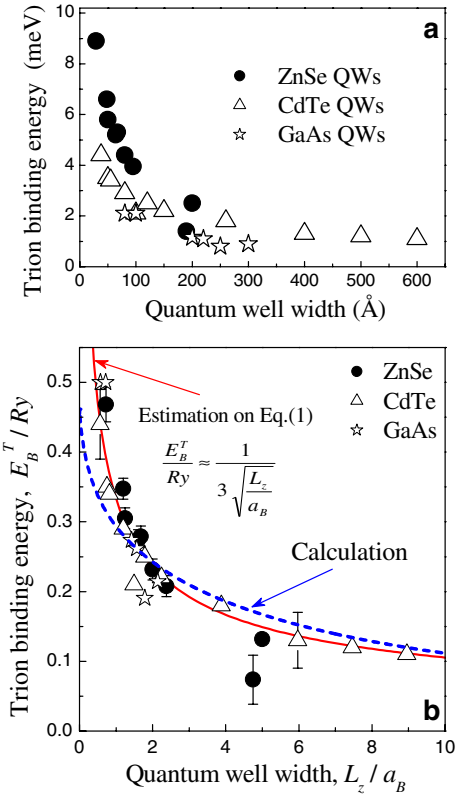

Fig. 1. The X^- trion binding energy E_B^T versus the QW width L_z plotted for different semiconductors: ZnSe by circles, CdTe by triangles, and GaAs by stars. These data are also collected in Table 1. (a) The experimental dependencies are plotted in natural units, i.e. energy and length being expressed in [eV] and [Å], respectively. (b) The experimental dependencies are plotted in 3D exciton units. The solid line represents the estimation (1) and the dotted line the calculation for the case of $\sigma = m_e/m_h = 0$.

3 Trion binding energy in ideal heterostructures

The fact that the experimental results for different semiconductors are tightly grouped around one curve seems to be rather surprising. Indeed, it signifies that the trion

binding energy in the most common semiconductor QWs can be easily estimated using only the value of well width and the parameters of the 3D exciton. However there is great amount of effects, which are not directly connected to these parameters but can also influence the trion binding energy. For example, electron-to-hole mass ratio, complex valence band and polaron effects are quite different in structures based on GaAs, CdTe and ZnSe, and some of them like built-in electric field or band offsets even vary from sample to sample. Nevertheless, the comparison of the normalized experimental values shown in Figure 1b does demonstrate that the scattering of the results caused by the variation of these effects in different semiconductor samples being relatively small. This implies that either the effects are neglible or their total contribution to the trion binding energy accidentally turns out to be the same in all considered heterostructures.

The simplest way for distinguishing these two cases is to evolve the common part of the binding energy, which should be universal for all semiconductors, and the contribution of such "sample-dependent" effects. For this task one should calculate the trion binding energy based on the quantum well width and the units of 3D exciton only. However, most of the available calculations [8–11] are basically related to the specifics of real heterostructures. Thus, at first we consider an universal model of the trion, which later is extended to particular samples. An advantage of such an approach is that in contrast to previous calculations the general model may be constructed in a most simplified and visual way.

The model is the following. The trion is treated as a three-body Coulomb system. The reduced mass and the permittivity are supposed to be isotropic and identical in QW and in the barriers. The real potential of the QW is replaced by an ideal one with infinite barriers. The hole is taken to be much heavier than the electron, so the mass ratio $\sigma = m_e/m_h$ is zero. In this case the hole occupies the center of QW, where the adiabatic potential of the electrons reaches a minimum [26]. A similar simplified approach was successfully used in [27] for X^- trion in magnetic field.

The Schrödinger equation for the trion in such a case is (hereafter we use 3D exciton units for the length and the energy):

$$\left[-\Delta_{r_1} - \Delta_{r_2} + (V_C(\mathbf{r_1},\mathbf{r_2}) + V^{QW}(z_1) + V^{QW}(z_2)) + E_B^X + E_B^T - 2E_e^{QW}\right]\Psi_T(\mathbf{r_1},\mathbf{r_2}) = 0. \quad (2)$$

Here $\mathbf{r_1}$ and $\mathbf{r_2}$ are three-dimensional vectors connecting the hole with the electrons, z_1 and z_2 are their projections on the growth direction. $V_C(\mathbf{r_1},\mathbf{r_2}) = 2(1/R - 1/r_1 - 1/r_2)$ is a Coulomb potential of the system, where $R = |\mathbf{r_1} - \mathbf{r_2}|$ is the distance between the electrons. E_B^X and E_B^T are the exciton and trion binding energies. $V^{QW}(z)$ is the quantum well potential, it is 0 if $|z| < L_z/2$ and $+\infty$ otherwise. E_e^{QW} is the quantization energy of the free electron in the ground state of the quantum well:

$$E_e^{QW} = \frac{1}{1+\sigma}\frac{\pi^2}{{L_z}^2} \qquad \left(\text{if } \sigma = 0,\ E_e^{QW} = \frac{\pi^2}{{L_z}^2}\right). \quad (3)$$

The equation (2) is solved by a variational method, when the energy in the ground state, calculated by a trial function with variational parameters, is minimized. We construct the simplest trial function with a minimum of variational parameters, which allows to estimate the trion binding energy for any value of quantum well width with sufficient accuracy needed for our purposes.

Let us notice, that we can calculate full correlation energies of the exciton and the trion only. The trion binding energy is by one order smaller, and the substraction can considerably increase the relative uncertanties. In order to minimize such numerical errors the exciton and trion energies should be calculated in the same manner. Thus, the trion function should be based on the exciton function, transforming to the latter when one of the electrons is removed. The similar approach we used in [28–30] for the analysis of the singlet and triplet states of trions in ideal two-dimensional quantum wells and for the calculation of the trion ground state in heterostructures with spatially separated carriers.

The simplest trial function for the exciton with only one variational parameter (a), which gives plausible results for the exciton binding energy in the whole range of the quantum well widths, is:

$$\Psi_X(\mathbf{r}) = A\,\exp(-a\,r)\,Z_0(z, L_z). \quad (4)$$

Here A is a normalization factor of the corresponding wave function. $Z_0(z, L_z)$ is the wave function of the ground state of the quantum well. In the case of the ideal quantum well it is:

$$Z_0(z, L_z) = \sqrt{\frac{2}{L_z}}\cos\left(\pi\frac{z}{L_z}\right), \quad \text{for } |z| \le L_z/2,$$
$$Z_0(z, L_z) = 0, \quad \text{for } |z| > L_z/2. \quad (5)$$

Here $\mathbf{r}$ is 3D vector connecting the hole and electron, and z is its projection on the growth direction. It is easy to see that function (4) turns into the exact wave function of the exciton in both limiting cases of an ideal 2D quantum well ($L_z \to 0$) and a 3D bulk semiconductor ($L_z \to \infty$).

Besides simplicity, the function (4) has one additional benefit. It is easy to show that the full kinetic energy of the electron in the case of any arbitrary quantum well potential $V^{QW}(z)$ is:

$$E_e^{kin} = \langle -\Delta_r \rangle = \left\langle \frac{\partial}{\partial r}\Psi_X(r,z) \mid \frac{\partial}{\partial r}\Psi_X(r,z) \right\rangle + E_e^{QW} - \langle \Psi_X | V^{QW} | \Psi_X \rangle = a^2 + E_e^{QW} - \left\langle V^{QW} \right\rangle. \quad (6)$$

Consequently, the quantization energy E_e^{QW} and the mean value of the quantum well potential V^{QW} in the Schrödinger equation can be eliminated analytically, which simplifies the calculations and let us avoid possible numerical mistakes. Thus, the binding energy of the exciton (E_B^X) can be estimated by the formula:

$$E_B^X = -\min_a(a^2 - \langle V_C(r)\rangle). \quad (7)$$

Here $V_C(r) = -2/r$ is a Coulomb potential between the electron and the hole. It should be noted, that the parameters of the quantum well are included in the mean value of the Coulomb potential through the last multiplier of the function (4). The equations similar to (6–7) are valid for all trial functions considered below.

The simplest trion function, based on the exciton function (4), is the 3-parameter Chandrasekhar-like one [31]:

$$\Psi_T(\mathbf{r_1}, \mathbf{r_2}) = A(\exp(-a_1 r_1 - a_2 r_2) + \exp(-a_2 r_1 - a_1 r_2))\,(1 + cR) Z_0(z_1, L_z) Z_0(z_2, L_z). \quad (8)$$

Here a_1, a_2, and c are variational parameters. In the same way as the function (5), (8) transforms into the appropriate Chandrasekhar's function in the limiting cases of two and three dimensions:

$$\Psi_T(\mathbf{r_1}, \mathbf{r_2}) = A(\exp(-a_1 r_1 - a_2 r_2) + \exp(-a_2 r_1 - a_1 r_2))\,(1 + cR). \quad (9)$$

The relative error in the trion binding energy obtained with the function (9) is known to be less than 10% both in the 2D and 3D cases. In intermediate quantum wells the function (8) efficiently takes into account the influence of the QW potential. Thus, we can expect that the estimations of the binding energies even in finite quantum well widths are also not far from the exact values.

The calculated trion binding energy versus QW width within the described approach is shown in Figure 1b (the dashed line pointed by $\sigma = 0$). The calculation is in surprisingly good agreement with the experimental data for wide quantum wells ($L_z \geq 2a_B$). Probably this means that the influence of effects specified by particular samples is relatively small and can be neglected. The other possible explanation is that these effects are fortuitously cancel each other in all considered heterostructures simultaneously. The out-of-order point for wide ZnSe QWs is explained by the possible presence of a build-in electric field, which causes a spatial separation of electrons and holes in growth direction. It has been shown [30] that the binding energy of the trion is quite sensitive to such a separation and rapidly decreases with electric field.

In narrow ($L_z < 2a_B$) quantum wells the experimental value of the binding energy sharply increases and the discrepancy with the calculations becomes remarkable. Most likely it can be concerned with the lateral localization of the trions on in-plane potential fluctuations or quantum well interface roughnesses [32]. Let us notice, that this contribution turns out to be more important in real heterostructures than the effect of the finite barriers, which leads to a decreasing of the binding energy and is considered below.

4 Corrections to the trion binding energy in real heterostructures

The good agreement between the experimental data and the general estimations obtained in the previous section signifies that the E_B^T variation caused by specific features of real heterostructures is small for $L_z \geq 2a_B$. In this paper we don't intend to consider all these features, which requires a more detailed approach. However, with the results of the previous section, we can roughly estimate the influence of some of them. Here we consider the role of electron-to-hole mass ratio, and the finite height of the QW barriers.

4.1 Correction due to the mass ratio scattering

To simplify the calculations we start by analyzing the exciton and then expand the obtained estimations to the trion case. In the case of $m_h \gg m_e$, the particle wave functions can be separated and the adiabatic approximation is applicable. The Schrödinger equation for the hole motion in the growth direction is (in 3D exciton units):

$$\left(-\frac{\sigma}{1+\sigma}\frac{\partial^2}{\partial z^2} + \left(V_e^{adiab}(z) + E_B^X - E_h^{QW}\right)\right) Z_h(z) = 0. \quad (10)$$

Here $Z_h(z)$ is the wave function of the hole in growth direction. $V_e^{adiab}(z)$ is a sum of the averaged Coulomb potential of the electron and the quantum well. E_h^{QW} is a quantization energy of the free hole in the ground state of the quantum well:

$$E_h^{QW} = \frac{\sigma}{1+\sigma}\frac{\pi^2}{{L_z}^2}. \quad (11)$$

As mentioned in the previous section, the quantum well potential is taken to be ideal, with infinite barriers. It is easy to show that the hole with infinitely heavy mass ($\sigma = 0$) is located in the minimum of the adiabatic potential ($z = 0$). The binding energy of the exciton in such case is:

$$E_B^X = -V_e^{adiab}(0). \quad (12)$$

As the mass ratio increases, the binding energy of the exciton decreases because, by the definition:

$$\left\langle Z_h \left| -\frac{\sigma}{1+\sigma}\frac{\partial^2}{\partial z^2} \right| Z_h \right\rangle \geq E_h^{QW}$$
$$\left\langle Z_h \left| V_e^{adiab}(z) \right| Z_h \right\rangle \geq V_e^{adiab}(0). \quad (13)$$

Qualitatively, if the mass of the hole becomes smaller, its localization length along z-direction increases until it achieves the width of the QW, and then stays unchanged. Therefore, the main factor, defining the evolution of the exciton binding energy with the mass ratio, is the hole localization in the growth direction due to the Coulomb attraction of the electron. The simplest wave function, taking this into account, is:

$$\Psi_X(r, z_e, z_h) = A\ \exp(-a\, r)\, Z_0(z_e, L_z) Z_0(z_h, (bL_z)). \quad (14)$$

Here r is the 3D distance between the particles, a is the reciprocal radius of the exciton, $b \in [0, 1]$ is the degree of hole localization. The value $b = 1$ means the function

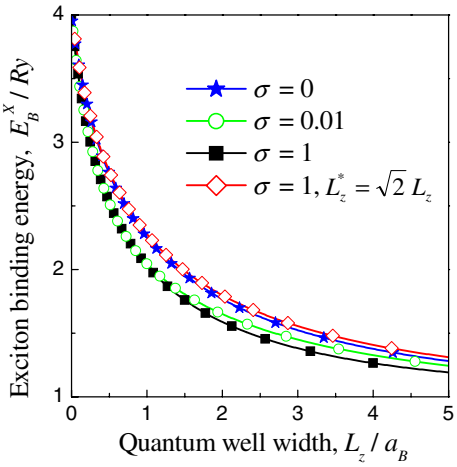

Fig. 2. The exciton binding energy E_B^X calculated as a function of the quantum well width L_z for different values of the mass ratio $\sigma = m_e/m_h$. The curves with $\sigma = 1$ (squares) and $\sigma = 0.01$ (circles) are nearly coincide. The curve with $\sigma = 0$ (stars) can be well approximated by the rescaled dependence with $\sigma = 1$ (diamonds), the coefficient being $\sqrt{2}$.

of the hole in the growth direction is nearly the same as that of the electron. The opposite case, $b = 0$, signifies that the hole is strongly localized in the center of the well corresponding to the case of an infinitely heavy hole.

The dependencies $E_B^X(L_z)$, calculated with the function (14) for few values of mass ratio $\sigma = m_e/m_h$ are shown in Figure 2. Let us see, that the difference in energy between even the extreme curves $\sigma = 0$ and $\sigma = 1$ is rather small (<10%) for all values of L_z. Moreover, it can be noticed that the curve corresponding to $\sigma = 0.01$ is closer to the curve for $\sigma = 1$ than $\sigma = 0$. For example, the curves $\sigma = 0.1$ and $\sigma = 1$ would not be distinguishable in the scale of the figure. This means, that the exciton binding energy is nearly independent on the mass ratio for $\sigma > 0.1$ and has an extremely weak dependence if $\sigma \in [0.1, 0.01]$. A considerable increase of E_B^X takes place only if the hole is unrealistic heavy ($\sigma < 0.01$). Consequently, we can neglect the variation of the binding energy of the exciton with the mass ratio for all experimental values of the latter.

Note, the curve with $\sigma = 0$ in Figure 2 can be well approximated by the curve with $\sigma = 1$ if the abscissa of the latter is multiplied by a coefficient $\sqrt{2}$. A small discrepancy takes place only for wide quantum wells, but even there, it is nearly negligible. This is a consequence of the fact, that the function (14) involves in an explicit form the electron and hole z-coordinates (z_e and z_h) only through the functions Z_0. Indeed, for the case of:

$$Z_0(z, L) = \frac{1}{\sqrt{\sqrt{\pi}L}} \exp\left(-\frac{z^2}{2L^2}\right), \qquad (15)$$

it can exactly be shown, that the binding energy of the exciton, given by equation (7), is the same both for $b \equiv 1$, $L \equiv L_z$ and $b \equiv 0$, $L \equiv \sqrt{2}L_z$:

$$E_B^X(L_z, \sigma = 1) \equiv E_B^X(\sqrt{2}L_z, \sigma = 0). \qquad (16)$$

The equation (16) is valid even if the exponent in the exciton function (14) is replaced by any other radial function. However, if the function Z_0 differs from a Gaussian function, the equality (16) becomes not valid. Nevertheless, as can be seen in Figure 2, it produces a good estimation of the binding energy of the light-hole exciton (i.e. with $\sigma = 1$) for a wide range of quantum well width values.

The results, obtained for the exciton, in some cases can be extended to the trion. As it is for the exciton, the binding energy of X^- trion, expressed in exciton units, is nearly independent of electron-to-hole mass ratio σ both for an ideal 2D quantum wells and for 3D bulk semiconductor [3]. The binding energy of the second electron in the trion is much smaller (∼10 times) than that of the first one. Therefore, in a rather crude model, the negatively charged trion, containing only one heavy particle, can be considered as an electron bound to an unperturbed exciton via some effective central potential. The latter depends on the mass ratio in such a way to keep the trion binding energy invariable in the limiting cases of 2D QW ($L_z \to 0$) and bulk semiconductor ($L_z \to \infty$).

In that way, the problem of the trion becomes very similar to the exciton one. Consequently, one can suppose that the only effect which causes an alteration of $E_B^T(L_z, \sigma)$ with σ is the increase of the exciton localization in the growth direction due to the interaction with the additional electron. By analogy to equation (16), the dependence of the trion binding energy with a mass ratio $\sigma = 1$ can be obtained via rescaling the curve with $\sigma = 0$:

$$E_B^T(L_z, 1) \approx E_B^T\left(\sqrt{2}L_z, 0\right), \qquad (17)$$

where the latter is known from the previous section. Obviously, all possible dependencies on the QW width of the trion binding energy are confined by these two extreme cases:

$$E_B^T(L_z, 0) \geq E_B^T(L_z, \sigma) \geq E_B^T(L_z, 1). \qquad (18)$$

The obtained scattering of the binding energy is less than 20%, which is even smaller than the dispersion of the experimental data. Moreover, as for the exciton binding energy, the trion E_B^T is expected to be about the same for $\sigma > 0.1$, allowing to take $\sigma = 1$ for any QW with realistic parameters. All these arguments prove the thesis, that the binding energy of X^- trion is nearly independent of the mass ratio in most quantum well heterostructures. The obtained estimation of $E_B^T(L_z, 1)$ is shown in Figure 3 by solid line.

It should be noted, that such a simplified picture of the trion is applicable in the 2D limiting case ($L_z \ll a_B$) only, where the motion in the growth direction can be omitted, or in wide QWs ($L_z \gg a_B$), where the exciton length is small compared to well width and the exciton can be treated as a single particle. In the intermediate case $L_z \sim a_B$ the movement of the electrons and holes in the growth direction should be considered independently and the approach might not work. However, the calculations of the trion binding energy in 250 Å-wide GaAs QW [25] show that there is also no strong dependence on the mass ratio.

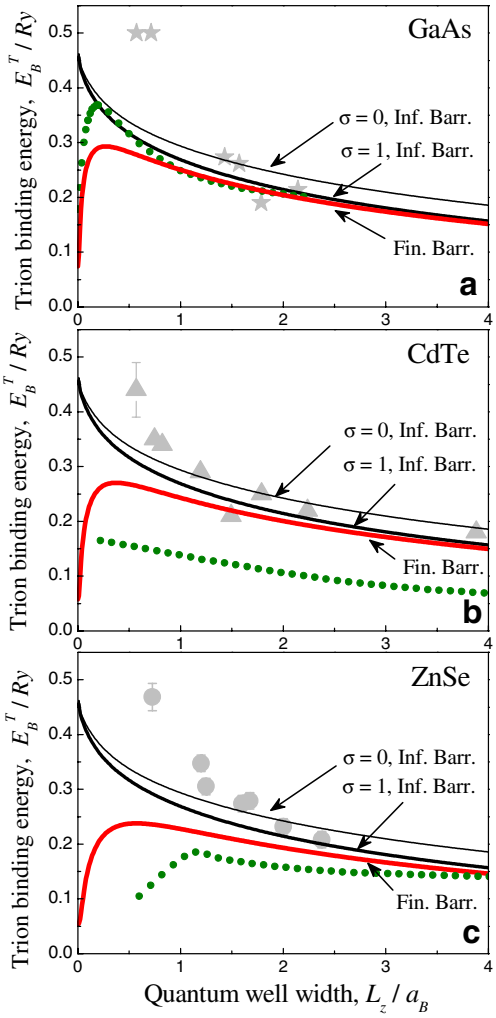

Fig. 3. The trion binding energy E_B^T versus the quantum well width L_z. The solid lines are calculations for $\sigma = 1$ with infinite barriers as well as with finite barriers and parameters of GaAs (a), CdTe (b) and ZnSe (c) QWs. The dotted lines are calculations for the same QWs adopted from [9]. The thin line is the dependence for $\sigma = 0$ given for a reference.

4.2 Corrections caused by finite barriers

In narrow QWs the penetration of the electron and hole wavefunctions into the barriers becomes important and the infinite barrier approximation tends to be inapplicable. However, in some crude form, it is possible to include this effect in our model without additional complicated calculations. For that, the real QW with the finite barriers should be considered as an ideal one with infinite barriers with an effective width for electrons (L_e) and holes (L_h) so that the trion binding energy remains unchanged. In wide quantum wells, where the penetration of the particle wavefunctions to the barriers can be neglected, one can take $L_e \approx L_h \approx L_z$. However, in narrow QWs, these values can be considerably different $L_e, L_h \neq L_z$. In particular, if $L_z \to 0$, $L_e, L_h \to \infty$. We choose the values of L_e and L_h in such a way so to keep the same mean-square deviation of the z coordinate in the ground state of a QW. It should also be noted, that a small relative inaccuracy in the effective well width does not lead to a considerable change of the binding energy. For example, if L_z is taken to be 10% larger, E_B^T decreases at most by 4%, which is quite small compared with the uncertainty of experimental data.

In most prevalent QWs, including our case, $L_h < L_e$, the simplifications can be kept. Indeed, as L_h decreases, the electron-hole interaction becomes stronger and the trion binding energy increases. However, the limiting case $L_h \to 0$ is very similar to the case when $\sigma = 0$, considered in the Section 3. Therefore, in the frames of our model, the simple estimation can be obtained:

$$E_B^T(L_e, L_e, 1) \leq E_B^T(L_e, L_e, \sigma) \leq E_B^T(L_e, L_h, \sigma) \leq E_B^T(L_e, L_h, 0) = E_B^T(L_e, L_e, 0), \quad (19)$$

where the relative difference between the boundaries does not exceed 20% (Fig. 1b).

Thus, for a simple estimation, we can take $L_h = L_e$ and extend the width L_z of a QW with finite barriers to the effective width L_e of an ideal QW with infinite barriers. Consequently, the calculation of a realistic dependence of $E_B^T(L_z)$ is reduced to the correct transformation of the abscissa axis.

The calculated dependencies for GaAs, CdTe and ZnSe QWs are plotted in Figure 3. In this figure we also show the results of previous calculations [9] converted to corresponding Bohr units. It is easy to see, that obtained estimations are in good agreement with reference [9] for GaAs QWs. However, some discrepancies are found for CdTe and ZnSe QWs. Nevertheless, it is necessary to emphasize that our simple and crude estimations allow to obtain a much better agreement with the experimental data then it has been achieved via the complicated calculations.

5 Conclusions

The experimental values of the trion binding energy (E_B^T) for various semiconductor quantum wells of different width (L_z), being represented in corresponding exciton units, are found to be well approximated by an universal function. The theoretical estimations confirm that in a simplified Coulomb model for wide QWs. In the case of very narrow QWs the calculated trion binding energy should be corrected towards higher value owing to lateral localization caused by inherent fluctuations of the QW width. Following experimental and theoretical analyses, the E_B^T is nearly independent of the electron-to-hole mass ratio at any value of quantum well width, and, for the sake of simplicity, calculations of the E_B^T can be performed with infinite hole mass values. The trion binding energy for QWs with finite barriers can be obtained from the universal dependence $E_B^T(L_z)$ for QWs with infinite barriers via some transformation. We believe that our findings allow to predict the trion binding energy in any semiconductor QW without additional complicated calculations.

This work was supported by the Deutche Forschungsgemeinschaft via Sonderforschungsbereich 410, Russian Foundation for Basic Research and the Federal Program on Support of Leading Scientific Schools. One of the authors (R.A. Suris) appreciates the support of the Alexander von Humboldt Foundation.

References

1. H.A. Bethe, Z. Phys. **57**, 815 (1929)
2. M.A. Lampert, Phys. Rev. Lett. **1**, 450 (1958)
3. B. Stebe, A. Ainane, Superlatt. Microstruct. **5**, 545 (1989)
4. K. Kheng, R.T. Cox, Y.M. d'Aubigne, F. Bassani, K. Saminadayar, S. Tatarenko, Phys. Rev. Lett. **71**, 1752 (1993)
5. G. Finkelstein, H. Shtrikman, I. Bar-Joseph, Phys. Rev. Lett. **74**, 976 (1995)
6. A.J. Shields, J.L. Osborne, M.Y. Simmons, M. Pepper, D.A. Ritchie, Phys. Rev. B **52**, R5523 (1995)
7. G.V. Astakhov, D.R. Yakovlev, V.P. Kochereshko, W. Ossau, J. Nürnberger, F. Faschinger, G. Landwehr, Phys. Rev. B **60**, R8485 (1999)
8. B. Stebe, G. Munschy, L. Stauffer, F. Dujardin, J. Murat, Phys. Rev. B **56**, 12454 (1997)
9. C. Riva, F.M. Peeters, K. Varga, Phys. Rev. B **61**, 13873 (2000)
10. L.C.O. Dacal, J.A. Brum, Phys. Rev. B **65**, 115324 (2002)
11. A.V. Filinov, M. Bonitz, Yu. E. Lozovik, Phys. Stat. Sol. (c) **0**, 1441 (2003)
12. J.M. Shi, F.M. Peeters, G.A. Farias, J.A.K. Freire, G.Q. Hai, J.T. Devreese, S. Bednarek, J. Adamowski, Phys. Rev. B **57**, 3900 (1998)
13. I.D. Mikhailov, F.J. Betankur, E.A. Escorcia, J. Sierra-Ortega, Phys. Stat. Sol. (b) **234**, 590 (2002)
14. J.M. Shi, F.M. Peeters, J.T. Devreese, Phys. Rev. B **51**, 7714 (1995)
15. V. Huard, R.T. Cox, K. Saminadayar, A. Arnoult, S. Tatarenko, Phys. Rev. Lett. **84**, 187 (2000)
16. G.V. Astakhov, D.R. Yakovlev, V.P. Kochereshko, W. Ossau, W. Faschinger, J. Puls, F. Henneberger, S.A. Crooker, Q. McCulloch, D. Wolverson, N.A. Gippius, A. Waag, Phys. Rev. B **65**, 165335 (2002)
17. O. Homburg, K. Sebald, P. Michler, J. Gutowski, H. Wenisch, D. Hommel, Phys. Rev. B **62**, 7413 (2000)
18. T. Wojtowicz, M. Kutrowski, G. Karczewski, J. Kossut, Acta Physica Polonica A **94**, 199 (1998)
19. V.P. Kochereshko, G.V. Astakhov, D.R. Yakovlev, W. Ossau, G. Landwehr, T. Wojtowicz, G. Karczewski, J. Kossut, Phys. Stat. Sol. (b) **220**, 345 (2000)
20. K. Kheng, K. Saminadayar, N. Magnea, J. Crys. Growth **184/185**, 849 (1998)
21. Z.C. Yan, E. Goovaerts, C. Van Hoof, A. Bouwen, G. Borghs, Phys. Rev. B **52**, 5907 (1995)
22. R. Kaur, A.J. Shields, J.L. Osborne, M.Y. Simmons, D.A. Ritchie, M. Pepper, Phys. Stat. Sol. (a) **178**, 465 (2000)
23. G. Finkelstein, H. Shtrikman, I. Bar-Joseph, Phys. Rev. B **53**, R1709 (1996)
24. A.J. Shields, M. Pepper, D.A. Ritchie, M.Y. Simmons, G.A.C. Jones, Phys. Rev. B **51**, 18049 (1995)
25. A. Esser, E. Runge, R. Zimmermann, W. Langbein, Phys. Rev. B **62**, 8232 (2000)
26. A.L. Efros, Sov. Phys. Semicond. **20**, 808 (1986)
27. M. Hayne, C.L. Jones, R. Bogaerts, C. Riva, A. Usher, F.M. Peeters, F. Herlach, V.V. Moshchalkov, M. Henini, Phys. Rev. B **59**, 2927 (1999)
28. R.A. Sergeev, R.A. Suris, Phys. Solid State **43**, 746 (2001)
29. R.A. Sergeev, R.A. Suris, Phys. Stat. Sol. (b) **227**, 387 (2001)
30. R.A. Sergeev, R.A. Suris, Semiconductors **37**, 1205 (2003)
31. S. Chandrasekhar, Astrophys. J. **100**, 176 (1944)
32. A.V. Filinov, C. Riva, F.M. Peeters, Yu. E. Lozovik, M. Bonitz, Phys. Rev. B **70**, 035323 (2004)